

# Riesz means and Greedy Sums.

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**Motivation.** The concept of greedy sum  $\sum_{s \in S} a_s$  of a numerical array  $\{a_s\}_{s \in S}$  was introduced in [1].

The direct product  $\{a_s b_t\}_{s,t \in S \times T}$  of two greedy summable numeric arrays  $\{a_s\}_{s \in S}$   $\{b_t\}_{t \in T}$  is not greedy summable in general. To sum up such products we adopt the general method of Riesz means.

**Arithmetic Riesz means.** Let  $\{\lambda_n\}$  be an increasing to infinity sequence of positive numbers and  $\kappa$  be a positive number. The series  $\sum a_k$  is called *summable* by Riesz method with parameters  $(\{\lambda_n\}, \kappa)$  to the sum  $A$  if

$$A = \lim_{\omega \rightarrow \infty} \sum_{\lambda_n < \omega} \left(1 - \frac{\lambda_n}{\omega}\right)^\kappa a_n, \quad (1)$$

If we consider a sequence of complex numbers  $a_n$ , such that  $|a_n| \searrow 0$ , then we can substitute in this definition  $\lambda_n = \frac{1}{|a_n|}$  and  $\omega = \frac{1}{\varepsilon}$ . Riesz means with such parameters being applied to Dirichlet series associated with a numerical array  $\{a_s\}_{s \in S}$  (see [1]) led us to the following definition. numeric array  $\{a_s\}_{s \in S}$  is called *greedy  $\kappa$ -summable* for some positive  $\kappa$ , if there exists the following limit

$$\lim_{\varepsilon \rightarrow 0} \sum_{|a_s| > \varepsilon} a_s \left(1 - \frac{\varepsilon}{|a_s|}\right)^\kappa, \quad (2)$$

which we name as *greedy  $\kappa$ -sum* of the array. For  $\kappa = 0$  the concept of greedy  $\kappa$ -sum converts into the original concept of greedy sum.

**Theorem 1.** *If an array  $\{a_s\}_{s \in S}$  has greedy  $\kappa$ -sum equal to  $A$  for some  $\kappa$  then it has the greedy  $\kappa'$ -sum for any  $\kappa' < \kappa$ .*

The above theorem follows from the theorem 16 of [2].

**Theorem 2.** *If an array  $\{a_s\}_{s \in S}$  has greedy  $\kappa_1$ -sum equal to  $A$  and another array  $\{b_t\}_{t \in T}$  has greedy  $\kappa_2$ -sum equal to  $B$ , then their product  $\{a_s b_t\}_{s,t \in S \times T}$  has greedy  $(\kappa_1 + \kappa_2 + 1)$ -sum equal to  $AB$ .*

The last theorem follows from the theorem 56 of [2].

## References

- [1] E. V. Shchepin *Greedy sum and Dirichlet series*, arXiv:1110.5285
- [2] G. H. Hardy, M. Riesz *The general theory of Dirichlet's series*, Cambridge University Press, London, 1915